ISSN 2229-5518

Optimal Mother Wavelet Function for EEG Signal Analyze Based on Packet Wavelet Transform

Hanan A. Akkar and Faris Ali Jasim

***Abstract*—**Electroencephalogram EEG signal is usually contaminated by different noise sources, which are called as artifacts. These artifacts need to remove before processing and analyzing the EEG signal. Several noise removal techniques are available and implemented. This paper presents a detail analysis of EEG de-noising using Packet Wavelet Transform (PWT). We carried out comparative study to choose the optimal mother wavelet basis function. The EEG database is freely acquired from MIT-BIH arrhythmia database for five control subjects. The EEG signal is grouped into five regional groups according to scalp regions. These groups are frontal, parietal, occipital, temporal, and central regions. Twenty five mother wavelet functions are researched. The functions are daubechies (db1 – db10), symlets (sym1 – sym10), and coiflets (coif1 – coif 5). These five groups are analyzed on these 25 mother wavelet functions (MWT). The cross correlation coefficients between signal of interest and wavelet de-noised signal are evaluated for the control subjects. The mother wavelets with better cross correlation coefficient can be selected as the optimal MWT basis function. This work is simulated and implemented using wavelet toolbox from MATLAB 2013 software environment.

***Index Terms*—** Electroencephalogram; wavelet; packet wavelet transform; cross correlation coefficients.

I. Introduction

Recording electrical activity along one's scalp is electroencephalogram EEG. HANS BERGER was the first person to measure it in 1929[1]. It's a noninvasive tool used to measure brain's electrical activity through electrodes placed on one's scalp. Potentials of the electrodes are boosted and registered as EEG (Electroencephalogram) signal. As a result, any abnormal activity of the brain and diagnosis of the
same can be done using (EEG) signals [2]. EEG signal consists of many spectral components. Amplitude of EEG signal for human beings is around 10 - 100 μV. Frequency range for EEG encompasses inconsistent upper and lower limits, though key frequencies are 0.1 to 30 Hz from
physiological standpoint. In normal individuals, brain waves are classified under one of four or five wave group

1) Delta (δ): All waves that fall under 3.5 Hz in EEG. These usually happen when the person is sleeping deeply in case of acute organic brain disease or during childhood.
2) Theta (θ): Theta waves are ones with frequency of 4-7Hz. These waves are common during childhood, or during periods of elevated stress in adults.
3) Alpha (α): Rhythmic waves that fall around the frequency range of 8-13Hz, and are common with normal individuals particularly during times of quiet, even though they remain awake or when they are thinking.
4) Beta (β): Low amplitude, though high frequency waves that ranges between 13-30Hz. The Beta waves are affected when mind is active.
5) Gamma (γ): EEG signals where frequency is > 30 Hz are called gamma waves [3].

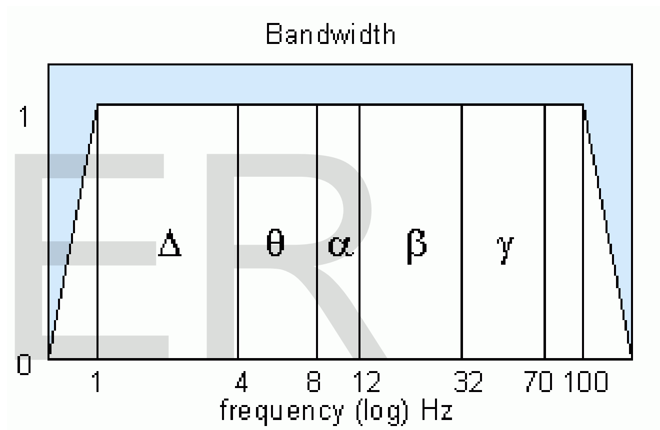

Fig. 1 Frequency Bands of EEG signal. [3]

EEG signals could be contaminated by noise easily, since it has very minor amplitudes. Noises could be anything including electrode noise or the ones created by the body and are termed artifacts. These should be eliminated from EEG signal, to analyze EEG signals accurately. Generally, non-physiological artifacts can compromise EEG data. In the human body there are sources that generate the physiological artifacts, like eye, heart or even muscles, and could cause ocular, cardiac or other artifacts caused by muscles. On other hand, there are technical sources that generate the non-physiological artifacts, which are due to environment or equipment. Before proper processing and analyzing the EEG signals, we should eliminate these artifacts from original EEG signal [4] [5].

Numerous de-noising methods were implemented to get rid of artifacts, as they affect EEG signal processing, directly. Adaptive filtering is applied by He *et al* [6] to remove ocular artifacts. Independent component ICA & adaptive filtering is deployed to minimize movement of eyes has been applied by Romero *et al* [7]. Novel adaptive method performed by Empirical Mode Decomposition (EMD) has been applied by Zeng *et al* [8]. Principal Component Analysis (PCA) incorporates a Math procedure, which derives several (probably) correlated variables and a number of uncorrelated variables termed as principal components has been applied by Dong Kang; Luo Zhizeng as a method to de-noise the EEG signal [9]. Kalman Filter (KF) has been employed by Shahabi

Prof. Dr. Hanan A. Akkar is with Department of electrical engineering. University of Technology, Iraq. (E-mail: dr_ hananuot@yahoo.com).
Faris Ali Jasim is with Department of electrical engineering. University of Technology, Iraq. (E-mail: faris2005@gmail.com).

for detection, and artifacts removal with good results [10].

Wavelets were established during the early 90s [11], and had many applications in signal processing. EEG is unstable signal and numerous studies that use adaptive wavelet thresholding algorithms were implemented to determine or eliminate noise [12]. Priyanka Khatwani [13] did a study to eliminate ocular artifacts deploying ICA, PCA and WT; deriving at a conclusion that wavelet method gave the most de-noising result due to its multiresolutional capacities. Wavelet transform analyses the signals in frequency and time domain and signals that have low noise amplitudes could be eliminated from signals by choosing the best wavelet to decompose the signal [14]. Removing features from sub-band of EEG signals through DWT is under review as a potential technique for analyzing EEG signal's characteristics [15].

The important step in wavelet analysis is the selection of WT basic function or the Mother Wavelet Transform (MWT). Numerous standard families with common WT basis functions are utilized, like Haar, Coiflets (coif), Daubechies (db) and Symlets (sym). A definitive selection of MWT basis function is still challenging, since WT basic functions properties and characteristic of EEG signal under that analyzed must be matched with precision [16]. Noor Kamal Al-Qazzaz [17] did a study to determine what the sufficient MWT for Discrete Wavelet Transform (DWT) to de-noise the EEG signals from 45 MWT basic functions, and she concluded that the "sym9" was the most compatible MWT functions with EEG using the DWT techniques. Also, Princy *et al*[18] suggested the future work to implement the Packet Wavelet Transform (PWT) for EEG de-noising which will give better results than DWT, since in DWT only low pass component of EEG signal is decomposed, while with PWT low and high pass components can be decomposed.

Therefore, in our paper, we did a study to determine optimal MWT basic function to remove artifacts from EEG signals through PWT. 25 MWTs, "daubechies (db1- db10), symlet (sym1 – sym10), and coiflets (coif1 –coif5)", were utilized to determine their suitability with EEG signals. Cross correlation method (xcorr) were utilized to match the resemblances of these MWT basic functions with the recorded EEG dataset. The options of optimal MWT are useful for decomposition, removing noise, feature extraction and reconstruction from EEG signal sub-bands [extracted from PWT] were used to understand the brain functions.

This paper is ordered in the following order: Methodology and Suggested Process are given under Section II, Test Simulation Results under Section III and Conclusions under Section IV of this paper.

II. METHODOLOGY

The general block diagram of the method proposed for determining optimal MWT among 25 functions is shown in Fig. 2.

A. *EEG signal acquisition*

The computer aided researchers has been used in their research the database available on line on the internet web sites. So the EEG database in our proposed approach is freely acquired from PHYSIONET MIT-BIH arrhythmia database [19]. The EEG signals were sampled at a sampling frequency of 256Hz. According to the scalp region, these EEG signals were grouped under five regions: the frontal region (*F*), temporal region (*T*), central region (*C*), occipital region (*O*), and parietal region (*P*). The length of each group signal is 2560 samples.

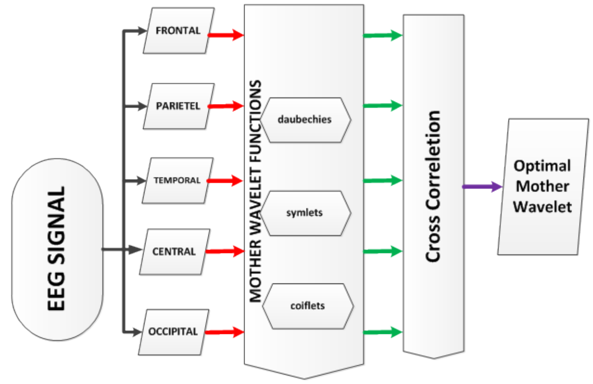

Fig. 2. The proposed block diagram

B. *Wavelet Transform (WT)*

The term "wavelet" is just a smaller wave. The smaller wave should have fast decay to zero and a minimum oscillation, in negative and positive directions, of their amplitude [20]. Wavelets are mathematical tools that could be used to get information from different types of data, which includes images and audio signals. Mathematically, wavelet is a function of zero average, which has energy that's potent in time. A family of wavelets has been constructed known as "Mother Wavelet" to be more pliable in extracting frequency and time information [14].

Wavelet transforms (WT) are signal-processing algorithms utilized to convert complex signals from frequency to time domains. However, unlike Fourier transforms, wavelets allow transformed data to be analyzed in both domains (frequency and time), simultaneously. Therefore the effective noise removal approach for non – stationary signals (EEG) is WT [21]. The WT of function f with reference to a listed mother wavelet $h(t)$ by dilation and translation is given by

$$W_f(a,b) = \int_{-\infty}^{\infty} f(t).h^*_{a,b}(t)dt \quad (1)$$

Here a and b are wavelet function parameters and $f(t)$ is the signal that should be transformed. The daughter wavelets are determined from one mother wavelet $h(t)$ by translation and dilation defined as

$$h_{a,b}(t) = \frac{1}{\sqrt{a}} h\left(\frac{t-b}{a}\right) \quad (2)$$

Where "a" is the scaling factor that controls the dilation or compression and "b" is the translation factor that determines the change in time. The constant $a^{\frac{1}{2}}$ is the energy normalization factor, included so that $\|h\| = \|h_{a,b}\|$ (i.e. keeps energy of the daughter wavelet = energy of the original mother wavelet, independent of "a" and "b"). To reconstruct the original function from its integral wavelet change the expression for the inverse wavelet transform is [20]:

$$f(t) = \frac{1}{C_h} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} W_f(a,b) h_{a,b}(t) \frac{\mathrm{dadb}}{a^2} \quad (3)$$

C. *Discrete Wavelets Transform (DWT)*

DWT for signal processing application is described in terms of filter banks. The EEG signal is first passing through low-pass filter to

ISSN 2229-5518

obtain approximate coefficients and then filtered using a set of high pass filters to obtain detailed coefficients. Therefore the signal is dividing up into spectral components called sub- band coding. This approach is called as the multi-resolution decomposition of the EEG signal. The main parameter of the wavelet is to choose the number of levels of signal decomposition where these levels are based on the dominant frequency components of the signal. The wavelet coefficients represent the energy distribution of EEG signal in frequency and time [22].

$$y_{\text{high}}[k] = \sum_n x[n].H[2k-n]$$
$$y_{\text{low}}[k] = \sum_n x[n].L[2k-n] \tag{4}$$

Here L: is for low pass filter, H: is for high pass filter, X[n]: represents the signal.

The similarity coefficient is further divided into detailed and approximation coefficients. By selecting the mother wavelet, the coefficients of these filter banks are determined. The indicated decomposition method is further repeated until the required frequency response is determined from the inferred input signal. Fig. 3 illustrates the general DWT tree with five decomposition levels [22].

The original signal sampling rate and the decomposition level directly determined the frequency band $\{f_m/2\text{: } f_m\}$ of every coefficients of DWT, that is ($fm = fs/2^{(l+1)}$), where f_s denotes sampling frequency, and l denotes level of decomposition.

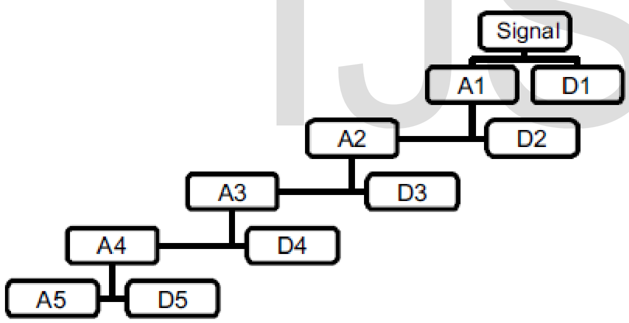

Fig. 3. Discrete wavelets transform [22].

D. *Packet Wavelets Transform (PWT)*

Wavelet packets are distinct linear combinations of wavelets forming bases that maintain most of their parent wavelets (i.e. orthogonally, smoothness, and localization properties). Wavelet packet transformation is applied to details and results approximations. EEG signal could be decomposed to low and high pass components, termed detail and approximations [13]. Fig. 4 illustrates the PWT decomposition tree.

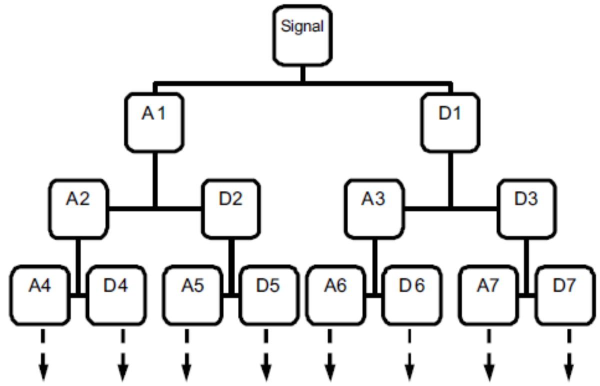

Fig. 4. Packet Wavelets Transform decomposition tree [22].

Each component in PWT tree could be seen as one filtered component that has a bandwidth of a filter that decreases with increasing level of decomposition, and the whole tree could be seen as a filter bank. The total sub bands are (2^L), and the bandwidth of each sub band at level "L" can be evaluated as:

$$\left[\frac{nfs}{2^{(L+1)}}, \frac{(n+1fs)}{2^{(L+1)}}\right] n = 0,1,2,\ldots\ldots..,2^L - 1\,; \tag{5}$$

Where: fs denote sampling frequency. So, the PWT analysis gives improved control of frequency resolution for decomposing the signal [23].

E. *Optimal Mother Wavelet Selection*

The first step in de-noising is process that uses wavelet transformation in selecting mother wavelet, which also has a set of functions (family of wavelets). Different common standards wavelet families are considered, that includes Symlets, Daubechies, Coiflets, Morlet Mexicanhat and Meyer wavelets. An important aspect in EEG signal processing through WT is choosing ideal decomposition level and MWT to minimize artifacts, which compromise EEG signals. The determination of the suitable MWT from wavelets family relies on their characteristic of similarity and orthogonality. The orthogonal families (Daubechies, Coiflets, and Symlets) used to obtain optimal reconstructed EEG signal [24].

In our study, we chose 25 MWTs of three varied orthogonal families, which includes Symlets (sym1 – sym10), Daubechies (db1-db10) and Coiflets (coif1-coif5). The chosen MWTs contained orthogonally properties needed to get approximate and detail coefficients from initial EEG signal without compromising information using PWT technique.

The cross correlation coefficients [17] between EEG signal of interest (X) and the wavelet de-noised signal (Y) is evaluated and stated as follows:

$$XCorr(X,Y) = \frac{\sum\left(X-\overline{X}\right)\left(Y-\overline{Y}\right)}{\sqrt{\sum\left(X-\overline{X}\right)^2\left(Y-\overline{Y}\right)^2}} \tag{6}$$

$\overline{X}$ & $\overline{Y}$ are mean value of X and Y, duly. The xcorr results of EEG signals grouped under five recording areas that correspond to the scalp region.

Threshold selection Signal de-noising using WT requires the following steps. First select the mother wavelet suitable for your signal as described above. Second choosing the number of decomposition levels. Third choose the optimal threshold for the obtained frequency filters. And finally reconstruct the de-noised signal at the last level. The number of levels for decomposition can be chosen according to the signal's dominant frequency.

In our analysis, the sampling frequency was selected to be 256Hz. Then the EEG signal was subjected to five decomposition levels through PWT. Therefore there are 32 sub bands frequencies extracted from the fifth level of PWT. The ΔF of each sub band was 4Hz according to this relation [25]

$$\Delta F = \frac{f_s}{2^{(L+1)}} \tag{7}$$

Table I illustrates the 32 sub bands decomposition frequencies. Methods for thresholding utilized with wavelet transform based filtering are to alter thus gathered coefficients. By deploying wavelet thresholding, noise in EEG signals can be eliminated. There are many ways to determine the right thresholding method and threshold values. Threshold is corresponding to the standard deviation of noise in the universal threshold T, and is construed as [26]

$$T = \sigma\sqrt{2lnN} \tag{8}$$

Where: N represents signal size and σ^2 represents noise variance, and calculated using

$$\sigma^2 = \frac{(median\ (|Xi|)}{0.645} \tag{9}$$

Where: ($|X_i|$) represents median value of absolute values of wavelet coefficients X_i. There are two types of thresholding and the threshold value can be evaluated as

$$Th = 1.416\ \sigma\ \left[Ln(L)\right]^{0.5} \tag{10}$$

1) Hard Thresholding

$$X_i = \begin{cases} X_i & for\ |X_i| \geq Th \\ 0 & for\ |X_i| \leq Th \end{cases} \tag{11}$$

2) Soft Thresholding

$$X_i = \begin{cases} \frac{Xi}{|Xi|}\left(|Xi| - TH\right) & for\ |X_i| \geq Th \\ 1 & for\ |X_i| \leq Th \end{cases} \tag{12}$$

In our study, SURE soft thresholding was utilized. The SURE threshold is a flexible soft thresholding method for finding threshold limit for every level, according to Stein's impartial risk estimation [27].

TABLE I
EEG SIGNAL DECOMPOSITION FOR 32 FREQUENCY BANDS

Sub band	Frequency band (Hz)	EEG bands
F0	0 – 4	Delta
F1	4 – 8	Theta
F2	8 – 12	Alpha
F3	12 – 16	
F4	16 -20	
F5	20 – 24	Beta
F6	24 – 28	
F7	28 – 32	
F8	32 – 36	
F9	36 -40	
F10	40 – 44	
F11	44 – 48	
F12	48 – 52	
F13	52 – 56	
F14	56 – 60	
F15	60 – 64	Gamma
F16	64 – 68	
F17	68 – 72	
F18	72 – 76	
F19	76 – 80	
F20	80 -84	
F21	84 – 88	
F22	88 – 92	
F23	92 – 96	
F24	96 – 100	
F25	100 – 104	
F26	104 – 108	
F27	108 – 112	Gamma
F28	112 – 116	
F29	116 – 120	
F30	120 – 124	
F31	124 – 128	

III. Results and Discussion

We are experimented our proposed PWT de-noising method in EEG signal derived from MIT-BIH database over 25 MWTs. This work is simulated and implemented using wavelet toolbox from MATLAB 2013 software environment. We've analyzed collected data to determine the ideal MWT for the scalp regions considering every part of the scalp as an averaged group over the 5 control subjects. We have de-noised each scalp region and the major variations with these 5 groups of scalp regions coupled with 25 MWTs were calculated utilizing xcorr as dependent variable for the control subjects. The xcorrs values are compared to determine the optimal MWT for every scalp region.

The results of MATLAB test shows that for frontal region, the optimal MWT is "db9" Fig. 5.

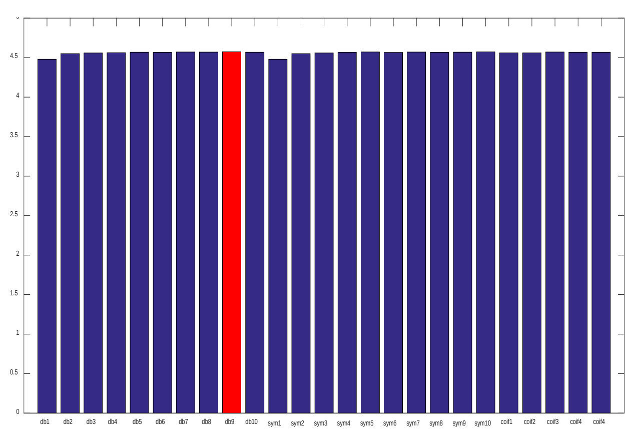

Fig. 5 Correlation coefficients of 25 mother wavelet PWT filter for frontal brain region.

In the parietal region, the highest xcorr that distinctly differs from other MWTs was for "sym9" Fig. 6.

ISSN 2229-5518

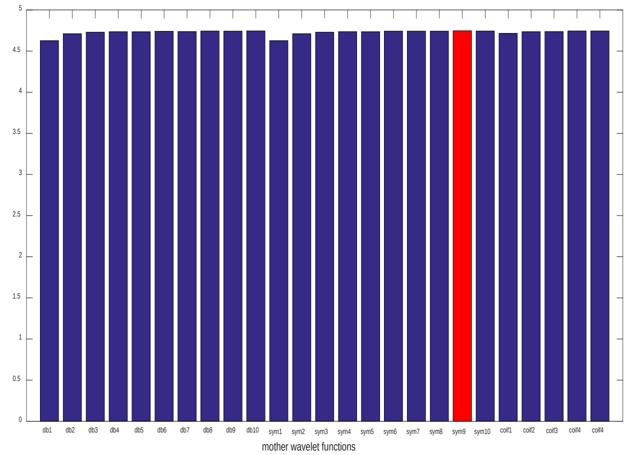

Fig. 6 Correlation coefficients of 25 mother wavelet PWT filter for parietal brain region.

The temporal region channels overlapped that of the parietal region, and the results were same, the highest dependent variable was for "sym9" Fig. 7.

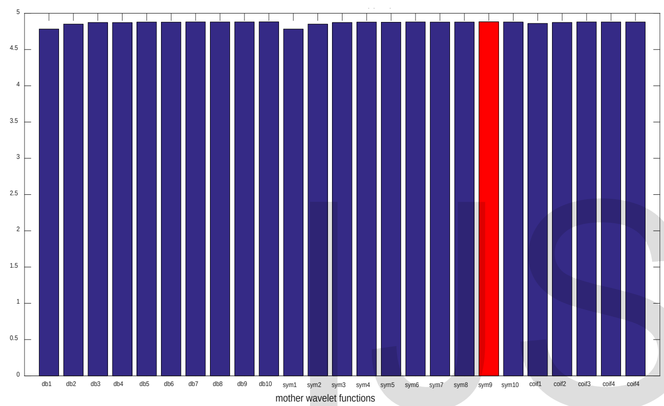

Fig. 7 Correlation coefficients of 25 mother wavelet PWT filter for brain's temporal region.

The occipital region channels the highest xcorr variable belo- ngs to "db 7" Fig. 8.

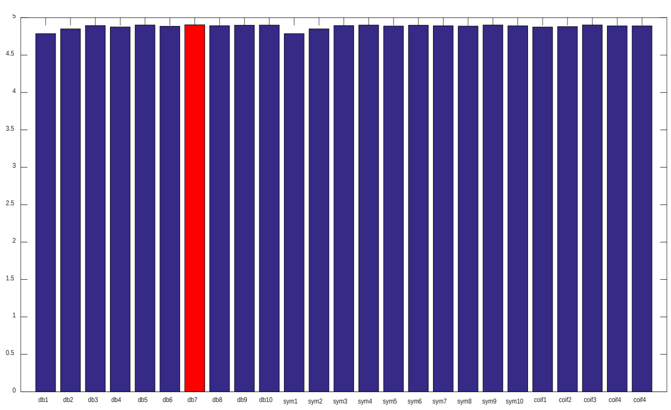

Fig. 8 Correlation coefficients of 25 mother wavelet PWT filter for the brain's occipital.

Furthermore, the highest xcorr coefficients for central region channels belong to "sym 9" which was grossly different from other MWTs Fig. 9.

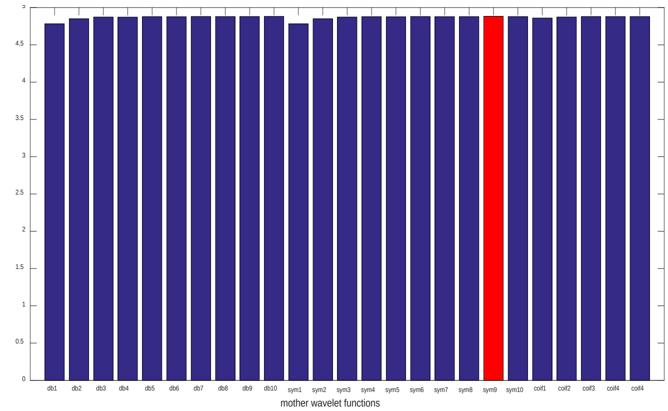

Fig. 9 Correlation coefficients of 25 mother wavelet PWT filter for brain's central region.

From Fig. 10, we can say that the " sym 9 " from Symlets family shows the mean highest compatibilities and similarities with EEG signal for said five scalp regions for the control subjects.

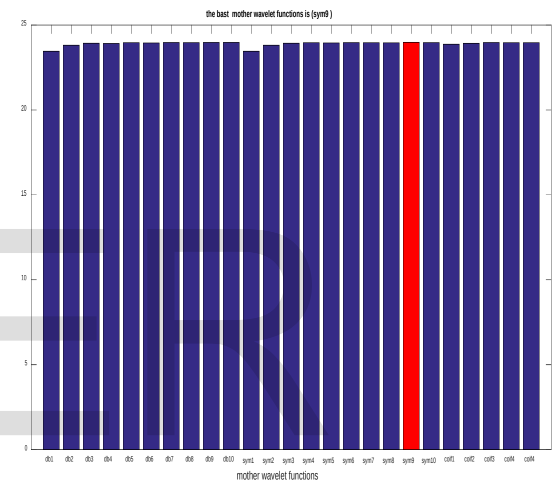

Fig. 10 Correlation coefficients of 25 mother wavelet PWT filter for 5 regions for 5 control subjects.

IV. CONCLUSION

EEG signal was de-noised using wavelet transform method. Wavelet transform analyses the signals in both frequency and time domain. Choosing the best wavelet to decompose the signal is important to remove the low noise amplitudes from the signals. Here, we are giving a thorough analysis of noise elimination and compression process of different wavelet families for EEG signal by applying Packet Wavelet Transform (PWT). The compatibility of 25 mother wavelet basis functions from the Symlets (sym1-sym10), Daubechies (db1-db10) and Coiflets (coif1-coif 5) orthogonal families were picked and analyzed due to the likeness to the five scalp regions of human brain for five control subjects. The selection of optimal mother wavelet basis function was based on the best cross correlation coefficient (xcorr) findings between the registered EEG signals and the packet wavelet transform noise elimination results.

REFERENCES

[1] R. Dhiman, J. Saini, Priyanka, and A. PMittal. "Artifact Removal From EEG Recordings," Presented at NCCI 2010, March, 2010, pp 62- 66.

[2] [2] V. Deepa , P. Thangaraj, and S. Chitra. (2010, June), "Investigating the performance improvement by sampling techniques in EEG data," International Journal on Computer Science and Engineering. vol. 2, pp. 2025-2028.

[3] S. Garg, and R. Narvey. (2013, June), " De-noising & Feature Extraction of EEG signal Using Wavelet Transform," International Journal of Engineering Science and Technology (IJEST). Vol. 5(6).
[4] C. Guerrero-Mosquera, A. Navia-Vazquez, and A. Trigueros. (2012), "EEG Signal Processing for Epilepsy;" INTECH Open Access Publisher: Morn Hill, Winchester, UK.
[5] W. Blume, M. Kaibara, G. Young, (2002), "Atlas of adult electroencephalography," Eur. J. Neurol., pp. 326–326.
[6] P. He, G. Wilson, C. Russell. (2004), "Removal of ocular artifacts from electro-encephalogram by adaptive filtering," Med. Biol. Eng. Computer, vol.42, pp. 407–412.
[7] S. Romero, M. Mananas, M. Barbanoj. (2009), "Ocular reduction in EEG signals based on adaptive filtering, regression and blind source separation," Ann. Biomed Eng . vol. 37, pp. 176–191.
[8] H. Zeng, A. Song, R. Yan, H. Qin. (2013), "EOG artifact correction from EEG recording using stationary subspace analysis and empirical mode decomposition," Sensors. vol. 13, pp. 14839–14859.
[9] D. Kang, L. Zhizeng. (2012, March), "A Method of De-noising Multi-channel EEG Signals Fast Based on PCA and DEBSS Algorithm," Presented at ICCSEE, vol.3, pp.322-326.
[10] Shahabi, Hossein, Moghim, Sahar, Zamiri-Jafarian. (2012, Oct.), "EEG eye blink artifact removal by EOG modeling and Kalman filter," Biomedical Engineering and Informatics (BMEI). pp.496-500.
[11] M. Ahmed, N. Shah, and S. Anka. (2016, Jan.), "Artifact Removal from EEG Recordings – An Overview," ARPN Journal of Engineering and Applied Sciences. vol. 11(1), pp. 502-510.
[12] V. Krishnaven, S. Jayaraman, S. Aravind, V. Hariharasudhan, K. Ramadoss. (2006), "Automatic Identification and Removal of Ocular Artifacts from EEG using Wavelet Transform," Measurement Science Review, vol. 6(4), Section 2.
[13] P. Khatwani, A. Tiwari. (2013, Feb.), "A survey on different noise removal techniques of EEG signals," International Journal of Advanced Research in Computer and Communication Engineering, Vol. 2(2).
[14] F. Ghawbar, M. Sami, N. Shah, and Y. Yousif. (2016), “PAT and Px Code Side lobe Reduction Using Wavelet Neural Network,” in Advances in Machine Learning and Signal Processing, ed: Springer, pp. 117-128.
[15] D. Übeyli. (2009), "Combined neural network model employing wavelet coefficients for EEG signals classification," Digit. Signal Process. vol.19, pp. 297–308.
[16] Amara Graps. (1995), "An Introduction to Wavelets," IEEE Computational Science and Engineering.
[17] N. Al-Qazzaz , S. Ali, S. Ahmad, M. Islam, and J. Escudero. (2015), "Selection of Mother Wavelet Functions for Multi-Channel EEG Signal Analysis during a Working Memory Task," Sensors, vol. 15, pp. 29015-29035.
[18] R. Princy, P. Thamarai, B. Karthik. (2015, March), " De-noising EEG Signal Using Wavelet Transform," Presented at International Journal of Advanced Research in Computer Engineering & Technology (IJARCET, vol. 4(3).
[19] Website: https://www.physionet.org/cgi-bin/atm/ATM
[20] S. Al-Shabkaan. (2004), “Wavelet Neural Network Approach for Recognition of Arabic Handwritten Character,” M.Sc. Thesis, University of Technology, Department of Electrical and Electronic Engineering, Iraq.
[21] V. Krishnaveni, S. Jayaraman, S. Aravind, V. Hariharasudhan, K. Ramadoss. (2006), "Automatic identification and removal of ocular artifacts from EEG using wavelet transform," Meas. Sci. Rev. Vol. 6, pp. 45–57.
[22] A. Subasi. (2007), "EEG signal classification using wavelet feature extraction and a mixture of expert Model," Expert Syst. Appl.vol.32, pp.1084–1093.
[23] G. Amiri, and A. Asadi. (2009, Dec.), " Comparison of Different Methods of Wavelet and Wavelet Packet Transform in Processing Ground Motion Records," International Journal of Civil Engineering. Vol. 7(4).
[24] J. Rafiee, M. Rafiee, N. Prause, M. Schoen. (2011), "Wavelet basis functions in biomedical signal Processing," Expert Syst. Appl.vol. (38), pp. 6190–6201.
[25] D. Safieddine, A. Kachenoura, L. Albera, G. Birot, A. Karfoul, A. Pasnicu, A. Biraben, F. Wendling, L. Senhadji ,and I. Merlet. (2012), "Removal of muscle artifact from EEG data: comparison between stochastic (ICA and CCA) and deterministic (EMD and wavelet-based) approaches," EURASIP Journal on Advances in Signal Processing.
[26] G. Kaushik, H. Sinha, L. Dewan. (2013, Dec.), "Biomedical Signals Analysis by DWT Signal De-noising with Neural Networks," International Journal of Recent Trends in Electrical & Electronics Eng.
[27] A. Nieslony. (2005, Feb.), "Wavelet-Based Methods for De-noising," adaptive thresholding 4-scale. , ch4.